Impact of Air Bubble Entrainment of Breaking Waves on Radiation Stress

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ABSTRACT

The current investigation is engaged with the discussion of radiation stress on allowing air bubble entrainment by breaking waves on an inclining sea shore in the surf zone. Beside of this, a vertical velocity component term has also been calculated letting air bubble effect, theoretically. To calculated wave set-up, the momentum balance equation has been modified by developing radiation stress term. Finally, the equation has been solved by using the Lax-Wendroff method of a progressive wave. It has been also checked a stability analysis for this method. The results demonstrate a good agreement with the experimental data and some popular model which is presented in the text.

1. INTRODUCTION

When waves propagate to the close shore zone, wave profiles turn out to be steeper and subsequent waves break. As a result, the air bubble entrainment is occurred into the breaking waves (Fig. 1). Breaking waves on beaches induce variations in radiation stress, driving longshore currents.

Radiation stress is the motion of energy, which is conveyed by the sea waves. At the point when these waves break, that energy is moved to the water segment, constraining nearshore flows [9]. Generally, the radiation stress assumed a significant function in different oceanographic wonder. For the clarification of different seaside confounded marvel, the radiation stress is required, particularly in breaking waves. For base bathymetric impacts in the close to shore the waves in the end steepen and breaks in the surf zone. At the point when wave is breaking, a thick tufts of air pockets makes and disseminates energy and force. Our comprehension of the breaking cycle itself is restricted, notwithstanding, to the absence of estimation.

Several researchers [6, 8, 10, 16] studied the surf zone hydrodynamics, but didn't incorporate air bubble for the breaking waves phenomenon. For random breaking waves, wave height and wave setup are calculated by [17]. Radiation stress and set-up in the near-shore region are calculated by [7], but breaking complexity is not included. A portion of the early attempts to contemplate the air bubble entrainment in the breaking waves in the surf zone

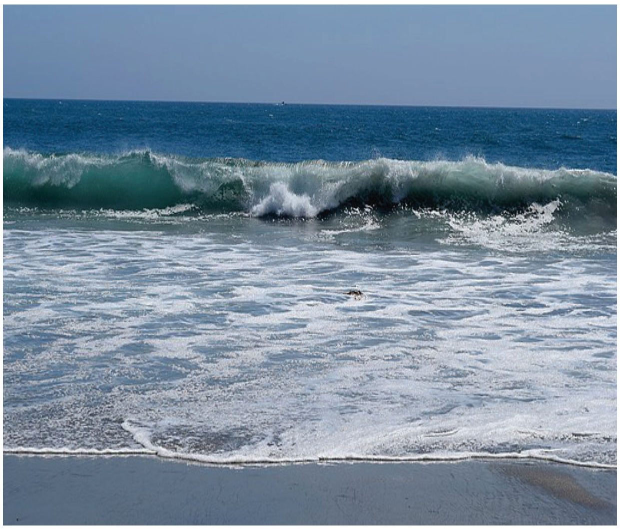

Fig. 1: Entrainment of air bubble in the breaking waves on a sloping beach.

are studied by [4]. At introductory stages, the entrained air bubble are answerable for dispersing the wave energy detailed in [18]. The reason for an abrupt decrease of wave stature and wave energy inside the surf zone could be entrained air rises into water guaranteed by [15]. A hypothetical investigation of the air bubble entrainment under the wind wave breaking in the sea surface layer had been summed up by [11]. An air bubble model proposed by [19] considering air bubble changes with water depth exponentially and checked by a series of laboratory data. Interaction of air and water and corresponding distribution

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is examined in the surf zone by [12]. As in [5] the storm wave height distribution is calculated, but the contribution of air bubble is neglecting. A dissipation model proposed by [3], taking air bubble effect linearly various with water depth but does not incorporate radiation stress for calculating wave set up in the surf zone.

For solving a system of nonlinear differential equation in hyperbolic or parabolic geometry numerically, [1,20] discussed the Lax-Wendroff scheme for better accuracy. Wave height and wave setup are calculated numerically by forward finite difference model which was found in [2, 18]. Linear water waves problems are solved by the Lax-Wendroff scheme that was described in [14].

Albeit various researchers have created various models dependent on various plans to figure out wave set-up in the surf zone, but still no one is included radiation stress considering air bubble effect. The present study concerns about this direction.

In this paper, the discussion is centered around developing radiation stress considering air bubble impact straightly fluctuates with water depth in the surf zone. Also, the vertical velocity component is calculated considering air bubble effect. Vertical velocity term can be used for better understanding of breaking waves in two dimensional study. The introduced plan incorporates void portion appropriations, air volume, modified vertical velocity component, radiation stress and momentum balance equation for computing wave setup. The wave set up of spilling and plunging breaking waves have processed in the surf zone by dissecting hypothesis and information. Finally, the results have been compared with the numerical and the experimental data.

2. MATHEMATICAL FORMULATION

2.1 Basic Assumption

Since [13] recommended that the void fraction dispersion fluctuates exponentially with water depth in the surf zone, yet from the perception of genuine field, void fraction circulation is likewise conceivable to consider that it changes straightly with water depth as in [3].

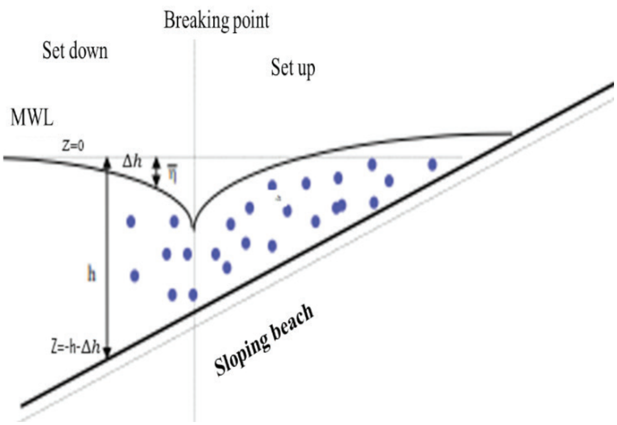

Fig. 2: Sketch of air bubble entrainment for breaking waves in the sloping beach.

The distribution of void fraction has been considered linearly varies with water depth (Fig. 2). The air bubble dispersion in the vertical way is given by the accompanying direct structure:

$$C(z) = \left(\frac{h + \Delta h + z}{h + \Delta h}\right) C_0 \tag{1}$$

where is the proportion of the volume locally involved via air bubble per unit width, h is the still water depth and C_0 indicates the reference concentration at the mean water surface $z = 0$.

The following boundary conditions are satisfied:

i) $C(z) = C_0$ at the mean water surface z = 0

& ii) $C(z) = 0$ at z = - h

2.2 Volume of Air

The ascent of the free-surface level Δh is an element of the measure of entrained air and water depth. The overall volume of entrained air into water per unit width is characterized as:

$$\Delta h = \int_{-h-\Delta h}^{0} C(z)dz \tag{2}$$

where z is letting vertically from the raised water surface. So that, from Eq. (2), we have

$$\Delta h = \frac{C_0 h}{2 - C_0} \tag{3}$$

which is the volume of the entrained air in the breaking water wave in the surf zone.

2.3 Modified Vertical Velocity Component Term

For the reason of air entrained in the water of the surf zone corresponding vertical component term *w* is changing. So that, we need to compute *w* for the case of air entrained in the water.

It is well known that the continuity equation for steady and compressible flow is,

$$\nabla.(\rho\vec{V}) = 0 \tag{4}$$

$$\Rightarrow \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial z}(\rho w) = 0 \tag{5}$$

But, for the reason of air entrained in the water, we have

$$\left.\begin{array}{l} u = u_w \\ w = w_w + w' \\ p = p_w \\ \rho = (1-C)\rho_w \end{array}\right\} \tag{6}$$

where, *w* denote the water and w' denote the correction term, ρ is the function of *z* only of the system. Using these values in the above equation and simplified, we get

$$\frac{\partial w'}{\partial z} - \frac{(w_w + w')}{(1-C)}\frac{C_0}{h+\Delta h} = 0 \tag{7}$$

with the help of Eq. (1),

$$\frac{\partial w'}{\partial z} + \frac{w'}{z+(h+\Delta h)(1-1/C_0)} = -\frac{w_w}{z+(h+\Delta h)(1-1/C_0)} \tag{8}$$

Integrating Factor of Eq. (8) is given by,

$$e^{\int \frac{1}{z+(h+\Delta h)(1-1/C_0)}}$$

Multiplying Eq. (8) by integrating factor, we obtain

$$w'\{z+(h+\Delta h)(1-1/C_0)\} = -\int w_w dz$$

$$= -\frac{\pi H}{T\sinh(kh)}\sin(\omega t - kx)\int \sinh k(h+z)dz \tag{9}$$

where, $w_w = -\frac{\pi H \sinh k(h+z)}{T\sinh(kh)}\sin(\omega t - kx)$

$$= -\frac{\pi H}{T\sinh(kh)}\sin(\omega t - kx)\frac{\cosh k(h+z)}{k} + c' \tag{10}$$

where, c' is the integrating constant.

Now, with the help of Eq. (1), we get

$$c' = -\frac{\pi H}{T\sinh(kh)}\sin(\omega t - kx)\frac{\cosh k(\Delta h)}{k}$$

Finally, we get

$$w' = \frac{p}{k}\left[\frac{\cosh k(\Delta h)}{z+(h+\Delta h)(1-1/C_0)} - \frac{\cosh k(h+z)}{z+(h+\Delta h)(1-1/C_0)}\right] \tag{11}$$

where, $p = \frac{\pi H}{T\sinh(kh)}\sin(\omega t - kx)$ (12)

Therefore, the vertical velocity component is

$$w = \frac{\pi H sin(\omega t - kx)}{T sinh\,(kh)}\left[\sinh(h+z) - \frac{1}{k\{z+(h+\Delta h)(1-1/C_0)\}}\{\cosh k(h+z) - \cosh k(\Delta h)\}\right] \tag{13}$$

2.4 Radiation Stress Considering Air Bubble Effect

Radiation stress is the overabundance stream of energy because of essence of the waves announced by [9]. The transition of force is included by two commitments: one because of wave – actuated speeds of the water particles and another because of the pressing factor. Presenting the air bubble impacts, the time found the middle value of radiation stress toward wave engendering is characterized as the time arrived at the midpoint of complete energy motion because of the presence of waves short the mean transition without waves which is written as:

$$S'_{xx} = \overline{\int_{-h-\Delta h}^{\eta}(p+\rho u^2)dz} - \int_{-h-\Delta h}^{0}(p_0 - \rho g\Delta h)dz \tag{14}$$

After simplification, we obtain

$$S'_{xx} = \overline{\int_{-h-\Delta h}^{\eta} pdz} + \overline{\int_{0}^{\eta} pdz} + \overline{\int_{-h-\Delta h}^{\eta} \rho u^2 dz} - \int_{-h-\Delta h}^{-\Delta h}(p_0 - \rho g\Delta h)dz \tag{15}$$

$$S'_{xx} = A + B + C + D \tag{16}$$

As the linear wave theory, we have the pressure term. Now, integrating this term by the above limit, we obtain

$$A = \frac{\rho_w g}{2}(h+\Delta h)^2 - \frac{\rho_w g H^2}{16} + \frac{\rho_w g H^2 kh}{8\sinh 2kh} \tag{17}$$

At the free surface p is almost equivalent to the static pressure factor beneath the free surface, which is reported by [9] as:

$$p = \rho_w g(\bar{\eta} - z) \tag{18}$$

So, the above eq. (18) simplified as:

$$B = \frac{\rho_w g H^2}{16} \tag{19}$$

We know, the elevation of water surface's is defined as

$$\bar{\eta} = \frac{H}{2}\cos(kx - \sigma t) \tag{20}$$

Again, with the help of Eq. (1), (6) and (20) and taking time average simplification, we obtain

$$C = \frac{\rho_w g H^2}{32\sinh 2kh}\{2kh + c_0 h(1+2k)\} \tag{21}$$

neglecting higher power of

And, using the static pressure, we obtain

$$D = -\frac{\rho_w g}{2}(h + \Delta h)^2 + \rho_w h \Delta h \tag{22}$$

Now, putting the value of *A, B, C* and *D* in Eq. (16) and using Eq. (3), we obtain

$$S'_{xx} = \rho_w g\left(\frac{C_0 h^2}{2 - C_0} + \frac{kh}{8\sinh 2kh} + \frac{H^2\{2kh + C_0 h(1+2k)\}}{32\sinh 2kh}\right) \tag{23}$$

which is the radiation stress in terms of air bubble effect. If, there is no air bubble is present then we obtain as [3] radiation stress formula.

2.5 Modified Momentum Balance Equation

The momentum flux is changed due to entrained air bubble, but the bottom pressure is fixed [3]. So that the momentum conservation equation is changed a

$$\frac{d\bar{\eta}}{dx} + \frac{1}{\rho_w g Q}\frac{dS'_{xx}}{dx} - \frac{R}{Q}\frac{dh}{dx} + \frac{S}{Q}\frac{dh}{dx} = 0 \tag{24}$$

where,

$$Q = \bar{\eta} + h - \frac{C_0 h}{2} - \bar{\eta}C_0 \tag{25}$$

$$R = \frac{C_0 h}{3} + \frac{\bar{\eta}C_0}{2} \tag{26}$$

and

$$S = \frac{C_0 h}{2} + \bar{\eta}C_0 \tag{27}$$

3. NUMERICAL PROCEDURE

In the earlier method, most of the model was solved by Finite Difference Method (FDM). But we have used to solve the governing eq. (24) by one dimensional Lax-Wendroff Method, which gives the better results of any other models [14]. Its accuracy up to 2nd order is the best.

3.1 Discretization

To determine wave set-up, we need to solve eq. (24) and the energy balance equation which was described by [14], simultaneously.

Applying the Lax-Wendroff scheme, the difference equation of eq. (24) be as:

$$\bar{\eta}_j^{k+1} = \bar{\eta}_j^k - \frac{\delta}{2\rho_w g Q}\{\acute{S}^k_{xx(j+1)} - \acute{S}^k_{xx(j-1)}\} + \frac{\delta^2}{2\rho_w g Q}\{\acute{S}^k_{xx(j+1)} - 2\acute{S}^k_{xx(j)} + \acute{S}^k_{xx(j-1)}\} - \frac{R}{2Q}(h^k_{j+1} - h^k_{j-1}) + \frac{\delta R}{2Q}(h^k_{j+1} - h^k_j + h^k_{j-1}) - \frac{S}{Q}(h^k_{j+1} - h^k_{j-1}) \tag{28}$$

where, $j = 1,2,3,\ldots\ldots\ldots n$, $S'^{k}_{x(j)}$ be the radiation stress including air bubble effect and $\delta = \frac{1}{2\Delta x}$.

In eq. (28), all of the quantities on the right hand side are known, so we can easily calculate the value of $\bar{\eta}_j^{k+1}$.

3.2 Stability Analysis

As in [14], the Von-Neumann stability analysis of the Lax-Wendroff scheme is satisfied the CFL (Courant-Frederic's - Lewy) condition.

So that, the Lax-Wendroff numerical scheme is courant stable for this model.

3.3 Initial Conditions

To calculate the eq. (28), there are two boundary conditions are required: (i) the offshore boundary (ii) the coastal boundary.

For this given situation, the data is required in the breaking zone as the starting zone to the shoreline:

(a) The wave height *H* and still water depth *h* at the close to the shore area.

(b) The bottom profile and wave period *T*.

(c) Moreover, the reference parameters, such as the void fraction C_0 = 0.25 and the free parameter $\beta = 0.8$ is considered for plunging breaker.

(d) C_0 = 0.25 and the free parameter $\beta = 1.17$ are for spilling breaker.

(e) Constant depth considered at the shoreline to avoid the undefined value of wave height as [14].

3.4 Computational Procedure

To calculate the wave set-up, the computation procedure is presented by the following flowchart which is described in Fig. 3:

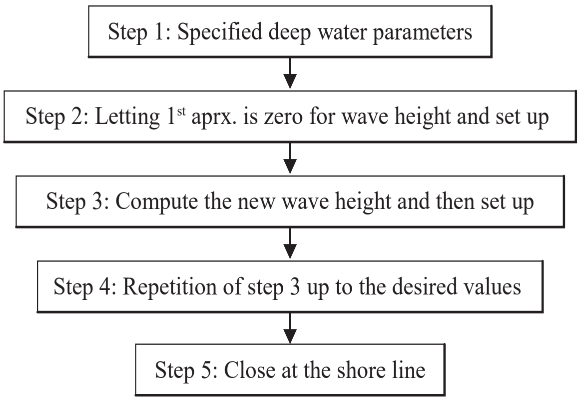

Fig. 3: Stream outline of the computational methodology.

4. RESULTS AND DISCUSSION

The regular waves including air bubble impacts is contrasted with the test results for sloping sea shore with a slope of 1/9.5 given by e [19]. The test was done in a 20 m-long, 0.80 m-wide and 0.60 m-depth wave flume.

The wave period was 1.12 s and 1.80 s for spilling and plunging breakers separately and loaded up with new water to a profundity of 0.40 m. Toward the finish of the wave flume, a wave safeguard was introduced to decrease reflection. Nitty gritty trials were given by [19], so we are not added here. The mathematical outcomes are confirmed with the down to earth information, standardized to their particular breaking esteems. Fig's. 8 and 9 shows the examination of wave setup which is determined by the force balance condition and then depicted by Eq. (24) at H0/L0 = 0.038 and H0/L0 = 0.076 for spilling and plunging breakers individually. As can be seen, the general understanding apparently is genuinely acceptable, albeit a slight variability in Fig. 8, because the data of its appearance in plunging breaker's. Fig's. 4, 5, 6 and 7 are showed that the comparison between present model and various popular model. A comparable variety can likewise be seen in different examinations, for example, [6] in Fig. 6. Two apparent reasons were noted by [6] and [10] for this inconvenience. As in, [1] first noted that after breaking point, no energy is dissipated until the curl touches down and white water appears. The effects of the roller in the breaking wave representation increases the momentum flux and therefore, the two effects may compensate each other mentioned by [6].

The results are showed excellent matched with the other results. The overall results of this developed model is found tremendous in the case of spilling breaking conditions; but, for plunging breaker, the obtain wave setup curve follows the little bit inconsistency at initial stage of start of wave set-up (Fig. 8).

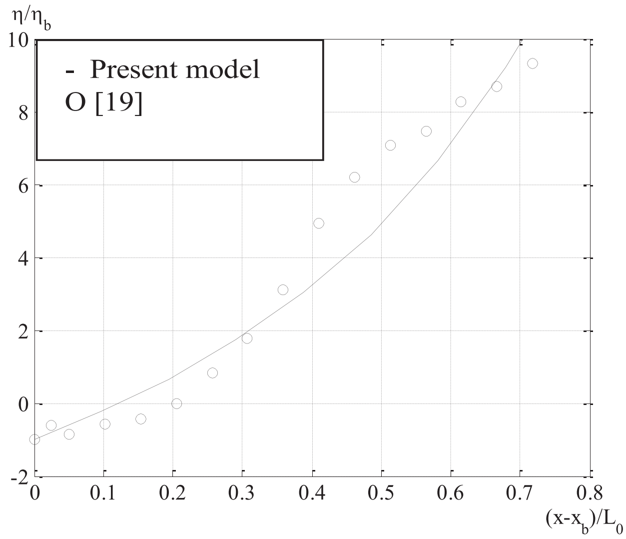

Fig. 4: Comparison of wave setup between present model and [19] for H_0/L_0 =0.062; β =1.5; C_0 = 0.20.

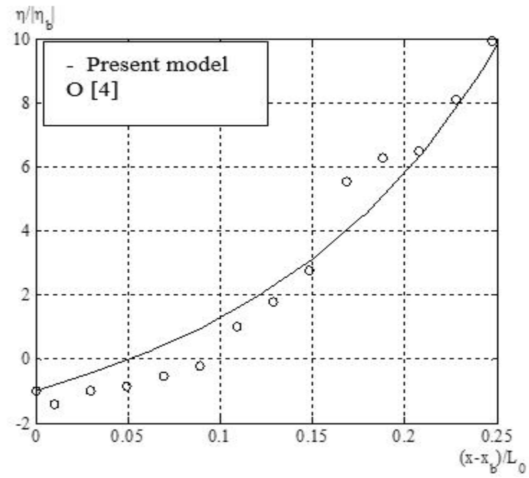

Fig. 5: Comparison of wave setup between present model and [4] for H_0/L_0 = 0.024; β = 0.80; C_0= 0.20.

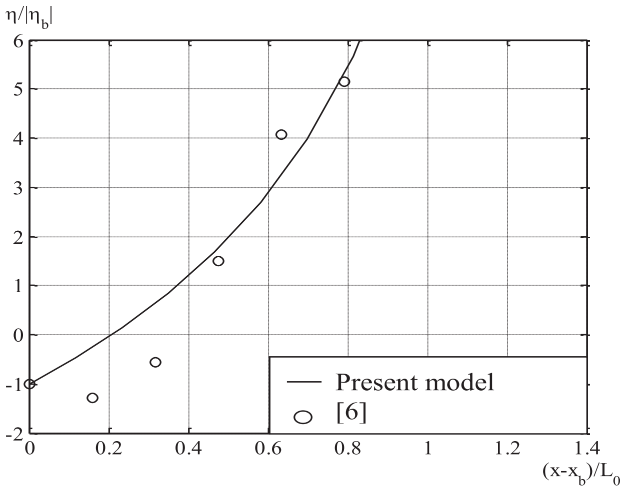

Fig. 6: Comparison of wave setup between present model and [6] for $H_0/L_0 = 0.062$; $\beta = 0.45$; $C_0 = 0.20$.

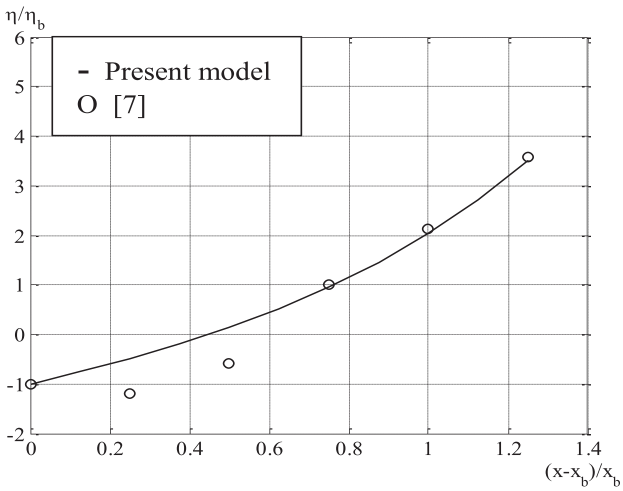

Fig. 7: Comparison of wave setup between present model and [7] for $H_0/L_0 = 0.062$; $\beta = 1.50$; $C_0 = 0.20$.

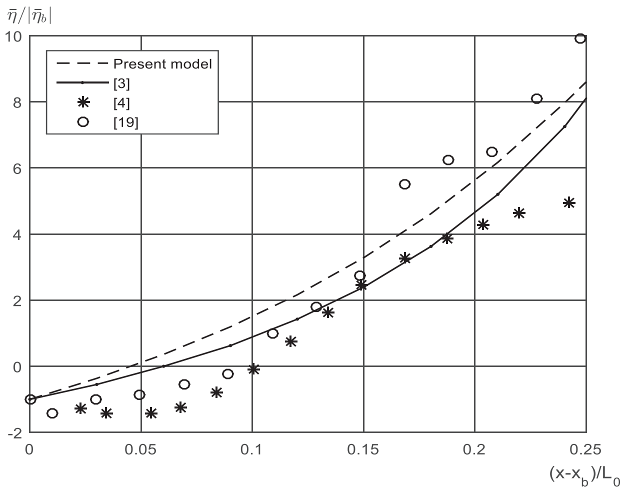

Fig. 8: Comparison of wave setup among various models and experimental data for $H_0/L_0 = 0.038$ (Plunging breakers condition).

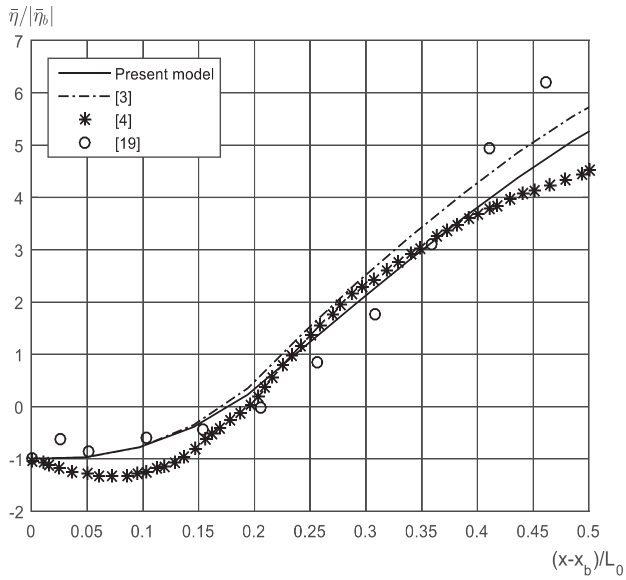

Fig. 9: Comparison of wave setup among present models, different models and experimental data for $H_0/L_0 = 0.076$ (Spilling breaker's condition).

5. CONCLUSION

The radiation stress and the vertical velocity component were developed theoretically considering the air bubble effect. Modified momentum balance equation was solved by Lax-Wendroof scheme and found good agreement with the experimental data and different models in the spilling breaker condition. In the plunging breaking waves little bit inconsistency was found, because the lag between early wave breaking and the initial wave set-up.

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